

Mark Scheme (Results)

January 2016

International Advanced Level in Core Mathematics C12 (WMA01/01)





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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 125.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:
- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.
- 3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod benefit of doubt
- ft follow through
- the symbol $\sqrt{\text{will be used for correct ft}}$
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- d... or dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper or ag- answer given
- L or d... The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the

subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.
- 8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

 $(x^{2} + bx + c) = (x + p)(x + q)$, where |pq| = |c|, leading to x = ...

 $(ax^2 + bx + c) = (mx + p)(nx + q)$, where |pq| = |c| and |mn| = |a|, leading to $x = \dots$

2. <u>Formula</u>

Attempt to use <u>correct</u> formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c$, $q \neq 0$, leading to $x = \dots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an <u>exact</u> answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these <u>may</u> not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does <u>not</u> cover this, please contact your team leader for advice.

January 2016 Core Mathematics C12 Mark Scheme

Question Number	Scheme	Marks
1(a)	$u_2 = 2 \times 2 - 6 = -2$, $u_3 = 2 \times (-2) - 6 = -10$ or $u_3 = 2 \times (2 \times 2 - 6) - 6 = -10$	M1 A1
		[2]
(b)	$\sum_{i=1}^{4} u_i = 2 + (-2) + (-10)$	M1
	+ (-26)	A1ft
	= -36	A1
		[3]
	Notes	5 marks
(a)	M1: Attempt to use the given formula correctly at least once. This may be implied by a correct	value for
(a)	u_2 or a value for u_3 which follows through from their u_2 or implied by correct answer for u_3	
	A1: u_3 correct and no incorrect work seen	
	A. <i>u</i> ₃ contest and no mean test work seen	
(b)	M1: Uses sum of the 3 numerical terms from part (a) (may be implied by correct answer for th Attempting to sum an AP here is M0.	eir terms).
	A1ft: obtains u_4 correctly (may be attempted in part (a)) and adds to sum of the first three term	ns from part
	(a) A1: -36 cao (-36 implies both A marks)	
	Special Cases:	
	Some candidates attempt $u_2 + u_3 + u_4 + u_5$ in part (b) – allow M1 only	
	Some candidates mis-copy one of their terms from part (a) into part (b) – allow M1 only	

Question Number		Scher	me		Marks
2(i)	Way 1: Way 2: Way 3: $\frac{49}{\sqrt{7}} = \frac{7^2}{7^{\frac{1}{2}}} = 7^{2-\frac{1}{2}}$ $7\sqrt{7} = 7^{1+\frac{1}{2}}$ $7\sqrt{7} = 7^{1+\frac{1}{2}}$ $7\sqrt{7} = 7^{1+\frac{1}{2}}$ $7\sqrt{7} = 7^{1+\frac{1}{2}}$ $7^a = \frac{49}{\sqrt{7}} \Rightarrow a = \frac{\log \frac{49}{\sqrt{7}}}{\log 7}$ $r^a = \frac{49}{\sqrt{7}} \Rightarrow a = \log_7 \frac{49}{\sqrt{7}}$ $r^a = \frac{49}{\sqrt{7}} \Rightarrow a = \log_7 \frac{49}{\sqrt{7}}$		M1		
	(<i>a</i> =	$=)1\frac{1}{2}$ (oe) or	see answer = $7^{1\frac{1}{2}}$	$7 = \sqrt{7} \Rightarrow u = \log_7 \sqrt{7}$	A1
			1		[2]
(ii)	Way 1: $\frac{10(\sqrt{18}+4)}{(\sqrt{18}-4)(\sqrt{18}+4)}$		(15√2	Way 2: $(15\sqrt{2}+20)(\sqrt{18}-4)$	
	$=\frac{\cdots}{2}$		$=15\sqrt{36}$ -	$-60\sqrt{2} + 20\sqrt{18} - 80$	B1
	$\frac{10}{\sqrt{18}-4} = 5\left(3\sqrt{2}+4\right) = 15\sqrt{2}$	$\sqrt{2} + 20*$		$\frac{50\sqrt{2} + 60\sqrt{2} - 80}{\frac{10}{18} - 4} = 15\sqrt{2} + 20*$	A1cso
					[3]
					5 marks
		Note			
(i)	Way 1:	V	Vay 2:	Way 3:	
	M1: Subtracts their powers of 7	and adds the	fraction to $7\sqrt{7}$ ir powers of 7 nswer only is 2 ma	M1: Correct use of logs t correct expression for <i>a</i> rks)	o obtain a
	_1		inexact decimals f $52 = 1.4999 \Rightarrow a$	or this mark e.g. a = 1.5 scores M1A0	
(ii)	Way 1: M1: Multiply numerator and deno $\sqrt{18} + 4$ or equivalent. The statem $\frac{10(\sqrt{18} + 4)}{(\sqrt{18} - 4)(\sqrt{18} + 4)}$ is sufficient b allow $\frac{10(\sqrt{18} + 4)}{\sqrt{18} - 4(\sqrt{18} + 4)}$ unless n brackets are implied by subsequen B1: Correctly obtains ± 2 in the det (Must follow M1 – i.e. treat as A implied by e.g. $\frac{10(\sqrt{18} + 4)}{18 - 16} = 5($ A1: Correct result with no errors s $\sqrt{18} = 3\sqrt{2}$ used before their fin Note that for Way 1, correct work $5\sqrt{18} + 20$ followed by $15\sqrt{2} + 2$	t work. nominator 1). May be $\sqrt{18} + 4$ een and al answer. leading to	at least 3 (not nee B1: All 4 terms of A1) A1: Obtains 10 v implied by e.g. 2	Way 2: expand $(15\sqrt{2} + 20)(\sqrt{18} - 10)(\sqrt{18} - 10)($.e. treat as $\sqrt{2}$ seen or

Question Number	Scheme	Marks
3.	$\int \left(6x - 3 - \frac{2}{\sqrt{x}} \right) dx = \frac{6x^2}{2} - 3x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + (c)$	M1 A1 A1
	$\left[\frac{6x^2}{2} - 3x - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + (c)\right]_1^4 = (28) - (-4) = 32$	M1 A1
		[5]
		5 marks
	Notes	
	M1: Attempt to integrate original $f(x)$ – at least one power increased $x^n \rightarrow x^{n+1}$ A1: Two of the three terms correct un-simplified or simplified (Constant not required) A1: All three terms correct un-simplified or simplified (Constant not required) M1: Substitutes limits 4 and 1 into their 'changed' function and subtracts the right way round A1: 32 cao (32 + c is A0) The question requires the use of calculus so a correct answer only scores no marks)	

Question Number	Scheme	Marks		
4.	$a + 3d = 3$ OR $\frac{6}{2}(2a + 5d) = 27$	M1 A1		
	$a + 3d = 3 \text{ OR } \frac{6}{2}(2a + 5d) = 27$ $a + 3d = 3 \text{ AND } \frac{6}{2}(2a + 5d) = 27$	A1		
	Eliminates one variable to find a or d from 2 equations in a and d	dM1		
	Obtains $a = 12$ or $d = -3$	A1		
	Obtains $a = 12$ and $d = -3$	A1		
		[6] 6 marks		
	Notes			
	M1A1: Writes down a correct (possibly un-simplified) equation for 4 th term or for sum of the first Allow the individual terms to be added for the sum e.g. $a + a + d + a + 2d + a + 3d + a + 4d + a + $ A1cao: A correct equation for 4 th term and a correct equation for the sum (allow either to be un-stable dM1: Eliminates one variable from two equations in <i>a</i> and <i>d</i> to find either <i>a</i> or <i>d</i> (see note below) A1: One variable correct (This implies previous M mark) A1: Both variables correct	5d = 27 simplified)		
	Note that if both equations are correct and there is no working and the values of <i>a</i> and <i>d</i> are both this scores dM0. Also if either or both equations is/are incorrect and values of <i>a</i> and <i>d</i> are obtain working this also scores dM0.			

Question number			Scheme				Marks
5(a)			through from O.	of a positive sin O with at least Condone diffe ow the x-axis.	one complet	e cycle	B1
			shown (shape with one from O to $\frac{3\pi}{2}$) $\frac{\pi}{2}$ and π		-	B1
			·				[2]
(b)	x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$		
	У	0	0.5	0.866	1		
	May be in	nplied by use o	Uses $\frac{1}{2} \times \frac{\pi}{12}$ of e.g. $\frac{1}{2}h = \frac{1}{2}$	$\left(\frac{\pi}{6} - \frac{\pi}{12}\right) = \frac{1}{2}$	(0.261)		B1
	$\dots \{(0+1) + 2(0.5 + 0.866)\}$			M1			
	0.4885176576 awrt 0.49				A1		
							[3] 5 marks
			N	otes			
(a)	Notes as above B1 : Correct shape with positive gradient through <i>O</i> B1 : Need not see endpoints labelled. Ignore any part of the curve to the left of the origin but if the curve extends beyond $x = \frac{3\pi}{2}$ then then $x = \frac{3\pi}{2}$ must be labelled on the diagram. Labels for $\frac{\pi}{2}$ and π may be on the diagram or in the text but not just in a table of values and must be in radians not degrees. (Allow awrt 1.57 and 3.14) The amplitudes must not be significantly different above and below the <i>x</i> -axis.					s for $\frac{\pi}{2}$ and	
(b)	B1: Need $\frac{1}{2}$ of $\frac{\pi}{12}$ or to see $\frac{\pi}{24}$ or $\frac{1}{2}$ of 0.261						
	 M1: requires first bracket to contain first plus last values and second bracket to include additional values from the two in the table. If values used in brackets are x values instead of y values this scores M0. A1: for awrt 0.49 				to include	e no	
	Separate trapezia may be used: B1 for $\frac{\pi}{24}$ and M1 for $\frac{1}{2}h(a + b)$ used 3 times						
	Special Case: Brack						,
	scores B1 M1 A0 u full marks can be gi Need to see trapezi	ven).	_			n done cor	rectly (then

Question Number	Scheme	Marks
6.	$f(x) = x^3 + x^2 - 12 x - 18$	
(a)	Attempts f(±3)	M1
	${f(-3)=} 0 \text{ so } (x+3) \text{ is a factor of } f(x).$	A1
		[2]
(b)	$x^{3} + x^{2} - 12x - 18 = (x+3)(x^{2} + \dots$	M1
	$x^{3} + x^{2} - 12x - 18 = (x+3)(x^{2} - 2x - 6)$ or $x^{3} + x^{2} - 12x - 18 = (x+3)(x - 1 + \sqrt{7})(x - 1 - \sqrt{7})$ oe	A1
		[2]
(c)	(x =) -3	B1
	$x = \frac{2 \pm \sqrt{4 + 24}}{2} = 1 \pm \sqrt{7} \text{ or by completion of square } (x - 1)^2 = 7 \text{ so } x = 1 \pm \sqrt{7}$ or $(x - 1 + \sqrt{7})(x - 1 - \sqrt{7}) = 0 \Rightarrow x = 1 \pm \sqrt{7}$	M1 A1
		[3]
		7 marks
	Notes	
(a)	M1: As on scheme – must use the <u>factor theorem</u> A1: for seeing 0 and conclusion which may be in a preamble and may be minimal e.g. QED, p tick etc. There must be no obvious errors but need to see at least $(-3)^3 + (-3)^2 - 12(-3) - 18 = 0$ for	
	invisible brackets e.g. $-3^3 + -3^2 - 12(-3) - 18 = 0$ provided there are no obvious errors.	
(b)	M1: Uses $(x + 3)$ as a factor and obtains correct first term of quadratic factor by division or an method e.g. comparing coefficients or finding roots and factorising	•
	A1: Correct quadratic and writes $(x+3)(x^2-2x-6)$ or $(x+3)(x-1+\sqrt{7})(x-1-\sqrt{7})$ of	
	Note that this work may be done in part (a) and the result re-stated here.	
(c)	B1: States -3 M1: Method for finding their roots. Allow the usual rules applied to their quadratic. This man finding the roots and not for just finding factors. You may need to check their roots if no shown e.g. if they give decimal answers (3.645, -1.645)	
	A1: need both roots. Correct answer implies M mark. Allow $x = \frac{2 \pm \sqrt{28}}{2}$	
	If they give extra roots e.g. $x = -3$, -1 , $\frac{2 \pm \sqrt{28}}{2}$, lose the final A mark (B1M1A0)	

Question Number	Scheme	Marks
7(a)	$(1+kx)^8 = 1 + {8 \choose 1}(kx) + {8 \choose 2}(kx)^2 + {8 \choose 3}(kx)^3 \dots$	M1
	$=1+8kx,+28k^2x^2,+56k^3x^3+$	B1, A1, A1
		[4]
(b)	Sets "56 k^3 " = 1512 and obtains $k^3 = \frac{1512}{56}$	M1 A1
	So <i>k</i> = 3	A1
		[3]
		7 marks
	Notes	
	term. The correct binomial coefficient needs to be combined with the correct power of x. Ign errors and omission of or incorrect powers of k. Accept any notation for ${}^{8}C_{2}$ or ${}^{8}C_{3}$, e.g. $\begin{pmatrix} 8\\2\\2\\8\\ \end{pmatrix}$ 28 or 56 from Pascal's triangle. This mark may be given if no working is shown, but either or both of $28k^{2}x^{2}$ and $56k^{3}x^{3}$ is B1: This is for 1 + 8kx and not for just $1 + \begin{pmatrix} 8\\1 \end{pmatrix} (kx)$ A1: is cao and is for $28k^{2}x^{2}$ or for $28(kx)^{2}$ A1: is cao and is for $56k^{3}x^{3}$ or for $56(kx)^{3}$ Any extra terms in higher powers of x should be ignored. Allow terms separated by commas or given as a list for all the marks.) or $\begin{pmatrix} 8 \\ 3 \end{pmatrix}$ or
(b)	M1: Sets their coefficient of $x^3 = 1512$ and obtains $k^n =$ where <i>n</i> is 1 or 3 A1: $k^3 = \frac{1512}{56}$ or equivalent e.g. 27 (May be implied by their final answer) A1: $k = 3$ cao (±3 is A0) Note (b) can be marked independently of part (a) so part (a) might be incorrect or not a they have $56k^3 = 1512$ etc. in (b)	ttempted but

Question Number	Scheme	Marks
	$7\sin x = 3\cos x$	
8 (a)	$(\tan x =)\frac{3}{7}$	B1
		[1]
(b)	$\tan\left(2\theta+30\right)=\frac{3}{7}$	B1ft
	$\tan^{-1} \frac{3}{7} (\alpha)$	M1
	One of θ = awrt 87 or awrt 177 or awrt 267 or awrt 357	A1
	Follow through any of their final θ 's for $\theta \pm 90n$ within range	A1ft
	All of $\theta = 86.6, 176.6, 266.6, 356.6$	A1
		[5]
		6 marks
	Notes	
(a)	B1: $(\tan x =)\frac{3}{7}$ or exact equivalent so accept recurring decimal (0.428571) but not r	ounded answer
(b)	B1ft: Correct equation as shown or follow through their value for tan <i>x</i> from part (a). Mu	ist be
	$\tan(2\theta+30) = \dots$ but $2\theta+30$ may be implied later by an attempt to subtract 30 and the	en divide by 2.
	If the processing is unclear or incorrect and $2\theta + 30$ is never seen, score B0 here.	
	M1: Finds arctan of their $\frac{3}{7}$. Could be implied by their value e.g. 23.19 or just \tan^{-1}	<u>.</u>
	A1: For one of either θ = awrt 87 or awrt 177 or awrt 267 or awrt 357	
	A1ft: Follow through any of their final answers to which an integer multiple of 90 has be	

Question Number	Scheme	Marks
9. (a)	$130000 \times (1.02) = 132600 *$ or $2\% = 2600$ and $130000 + 2600 = 132600 *$	B1
(b)		[1]
(b)	(<i>r</i> =) 1.02	B1 [1]
(c)	Uses $130000 \times (1.02)^{N-1} > 260000$ or $130000 \times (1.02)^{N-1} = 260000$	M1
	So $(1.02)^{N-1} > 2$	A1
	$(N-1)\log_{10}(1.02) > \log_{10} 2$ or $(N-1)\log_{10}(1.02) = \log_{10} 2$ or $(N-1) > \log_{1.02} 2$ or $(N-1) = \log_{1.02} 2$	M1
	$N > \frac{\log_{10} 2}{\log_{10} (1.02)} + 1^*$	Alcso
		[4]
(d)	(N =) 37	B1
		[1]
		7 marks
	Notes	
(a)	B1: A reason must be provided for this mark as the answer is printed. Allow both $120000 \times (1 + 2\%)$ and $120000 \times (102\%)$ as both give the correct answer when ont	tarad this
	Allow both $130000 \times (1+2\%)$ and $130000 \times (102\%)$ as both give the correct answer when ent way on a calculator. But not $130000 \times 1+2\%$	lei eu tills
(b)	B1: For 1.02 of e.g. allow $\frac{51}{50}$	
(c)	M1: Correct inequality or equality – may use <i>r</i> or their <i>r</i> or 1.02 and may use <i>N</i> or <i>n</i> . A1: $(1.02)^{N-1} > 2$ cao. Allow $(1.02)^{n-1} > 2$	
	M1: Correct use of logs power rule on their previous line which must have come from using the of a GP. Condone missing brackets for this mark e.g. $N - 1\log_{10}(1.02) > \log_{10} 2$. (May follow use	use of =
	instead of > or use of r instead of 1.02 or use of N instead of $N - 1$). These cases can get MOAON the base to be absent or just 'ln' for this mark. If the inequality sign is reversed at this point, still M1.	
	A1*: Answer is exactly as printed (including the bases) and all inequality work should be correprevious marks scored and no missing brackets earlier. Allow this mark to score from a correct line provided the power rule is used. So fully correct work leading to	
	$(N-1)\log_{10}(1.02) > \log_{10}2 \Longrightarrow N > \frac{\log_{10}2}{\log_{10}(1.02)} + 1$ scores the final M1A1 but	
	$(1.02)^{N-1} > 2 \Longrightarrow N > \frac{\log_{10} 2}{\log_{10} (1.02)} + 1$ scores M0A0 (no explicit use of power rule)	
(d)	B1: Only need $N = 37$ – may follow trial and error or uses logs to a different base. Do not allow $N \ge 37$ or $N > 37$ or $N = 37.0$	

Question Number	Scheme	Marks
	$y = 12x^{\frac{5}{4}} - \frac{5}{18}x^2 - 1000$ $\frac{dy}{dt} = 12 \times \frac{5}{4}x^{\frac{1}{4}} - \frac{10}{18}x$	
10.(a)	$\frac{dy}{dx} = 12 \times \frac{5}{4} x^{\frac{1}{4}} - \frac{10}{18} x$	M1 A1
(b)		[2
(b)	Put $12 \times \frac{5}{4} x^{\frac{1}{4}} - \frac{10}{18} x = 0$ so $x^n = k (n \in \Box, k \neq 0)$	M1
	$\therefore x = ()^{\frac{4}{3}}$ $\therefore x = 81$	dM1
	$\therefore x = 81$ (Ignore x = 0 if given as a second solution)	A1
	So $y = 12(81)^{\frac{5}{4}} - \frac{5}{18}(81)^2 - 1000$ i.e. $y = 93.5$	dM1A1
(2)	-2	[:
(c)	$\frac{d^2 y}{dx^2} = -\frac{15}{4}x^{-\frac{3}{4}} - \frac{5}{9}$	B1ft
	Substitutes their non-zero x (positive or negative) into their second derivative.	M1
	Obtains maximum after correctly substituting 81 into correct second derivative to give correct	
	negative quantity $-\frac{15}{36}$ o.e. or decimal e.g0.4 (see note below) and considers negative	
	sign deducing maximum.	A1
	Note that a correct second derivative followed by $x = 81 \Rightarrow \frac{d^2 y}{dx^2} = \frac{15}{4} 81^{-\frac{3}{4}} - \frac{5}{9} = -\frac{5}{12}$ therefore	
	maximum scores B1M1A0 here.	
		[. 10 man
	Notes	10 mark
(a)	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 \to 0) A1: Two correct terms and no extra terms. Terms may be un-simplified.	
(a) (b)	M1: Attempt to differentiate – power reduced by one $x^n \rightarrow x^{n-1}$ (but not just 1000 \rightarrow 0)	
	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 \to 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where <i>n k</i> is non-zero dM1: Correct processing to obtain a value for <i>x</i> . (Dependent on the first method mark). This may only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must	ark can
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	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 \to 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where n k is non-zero dM1: Correct processing to obtain a value for x . (Dependent on the first method mark). This may only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of x . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} (a - bx^{\frac{3}{4}}) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their positive value for x into $y = \dots$ and not into $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao	ark can have the $x = \sqrt[3]{k}$ e first
	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just $1000 \to 0$) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where n k is non-zero dM1: Correct processing to obtain a value for x . (Dependent on the first method mark). This may only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of x . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} \left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their positive value for x into $y = \dots$ and not into $\frac{dy}{dx} = \dots$ (Dependent on the method mark)	ark can have the $x = \sqrt[3]{k}$ e first
(b)	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 \to 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where n k is non-zero dM1: Correct processing to obtain a value for x . (Dependent on the first method mark). This must only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of x . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} \left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their positive value for x into $y = \dots$ and not into $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao If $x = 81$ appears from no working following a correct derivative score M1M0A0 then allow fur B1ft: Correct follow through second derivative M1: Substitutes their non-zero x (positive or negative) into their second derivative. Note: Solving $\frac{d^2y}{dx^2} = 0$ is M0	ark can have the $x = \sqrt[3]{k}$ e first <u>ill recovery</u>
(b)	M1: Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just 1000 \to 0) A1: Two correct terms and no extra terms. Terms may be un-simplified. M1: Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where n k is non-zero dM1: Correct processing to obtain a value for x . (Dependent on the first method mark). This must only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must correct powers of x . E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}} \left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. A1: cao dM1: Substitutes their positive value for x into $y = \dots$ and not into $\frac{dy}{dx} = \dots$ (Dependent on the method mark) A1: cao If $x = 81$ appears from no working following a correct derivative score M1M0A0 then allow fur B1ft: Correct follow through second derivative M1: Substitutes their non-zero x (positive or negative) into their second derivative.	ark can have the $x = \sqrt[3]{k}$ e first <u>all recover</u>

Question Number	Scheme	Marks
11(a)	$16^2 = 10^2 + 12^2 - 2 \times 10 \times 12 \cos \angle YXZ$	M1
	$\cos \angle YXZ = \frac{10^2 + 12^2 - 16^2}{2 \times 10 \times 12}$ or $\frac{-12}{240}$ or -0.05	A1
	$\angle BOC = 1.62(08)$ (N.B. 92.87 degrees is A0)	A1
A \		[3]
(b)	Uses $s = 5\theta$ with their θ from part (a)	M1
	awrt 8.1	A1
	Perimeter = $r\theta + 28$, = 28 + their arc length	M1
	awrt 36.1	A1
		[4]
(c)	area of sector $=\frac{1}{2}(5)^2\theta$	B1ft
	area of triangle = $\frac{1}{2}$ 10×12sin θ (= 59.92 or 59.93)	B1ft
	Area of shaded region = $\frac{1}{2} \times 10 \times 12 \sin \theta - \frac{1}{2} (5)^2 \theta = 59.9 20.2$ = 39.7 (cm ²)	M1 A1
		[4]
		(11 marks)
	Notes	
(a)	M1: Uses cosine rule – must be a correct statement	
	A1: Correct value or correct numerical expression for $\cos \angle YXZ$	
(b)	A1: accept awrt 1.62 and must be seen in part (a) (answer in degrees is A0 (92.865)) M1: Uses $s = 5\theta$ with their θ in radians, or correct formula for degrees if working in degrees	
(0)	A1: Accept awrt 8.1 (may be implied by their perimeter)	
	M1: Adds their arc length to 28 or $(16 + 7 + 5)$	
	A1: Accept awrt 36.1 do not need units (ignore any given)	
(c)	B1ft: This formula used with their θ in radians or correct formula for degrees	
	B1ft: Correct formula for area used – may use half base times height (may be implied by a cor	rect answer
	(59.9))	
	M1: Subtracts their sector area from their triangle area this way round .	
	A1: awrt 39.7 – do not need units (ignore any given) Alternative approach to finding angle YXZ and area of triangle:	
	Let foot of perpendicular from X to YZ be W and $XW = h$ and $YW = x$ so $WZ = 16 - x$:	
	$h^{2} + x^{2} = 100, h^{2} + (16 - x)^{2} = 144 \implies x = \frac{53}{8}, h = \frac{3\sqrt{399}}{8}$ M1: Correct work leading to values of	f x and h
	$\angle YXZ = \sin^{-1}\left(\frac{53}{80}\right) + \sin^{-1}\left(\frac{25}{32}\right) = 1.62$ A1:Correct expression for $\angle YXZ$, A1: awrt 1.62	
	The B1 for the triangle area in (c) can then score for $\frac{1}{2} \times 16 \times \left\ \frac{3\sqrt{399}}{8}\right\ $. Note this is $3\sqrt{399}$	

Question Number	Scheme	Marks
	(a) and (b) can be marked together	
12(a)	$f(x) = \frac{16 + 24\sqrt{x} + 9x}{x}$	M1
	$f(x) = 16x^{-1} + 24x^{-\frac{1}{2}} + 9$	M1A1A1
(b)	$f'(x) = -16x^{-2} - 12x^{-\frac{3}{2}}$	[4] [4] [4]
		[2]
(c)	When $x = 4$, $y = 25$	B1
	f'(4) = $-1 - \frac{12}{8} = -2\frac{1}{2}$ Equation of tangent is $y - 25 = -\frac{5}{2}(x - 4)$	M1
	Equation of tangent is $y-25 = -\frac{5}{2}(x-4)$	M1 A1
		[4
		10 marks
	Notes	
	M1: Divides at least one term in numerator by x correctly following an attempt at expansion $\frac{16}{x}$. A1: Two correct terms A1: All terms correct	i <u>sion</u> . May just be
(b)	M1: Evidence of differentiation $x^n \to x^{n-1}$ of an expression of the form Ax^{-1} or Bx^k s	o $x^{-1} \rightarrow x^{-2}$ or
	$x^{k} \rightarrow x^{k-1}$ $(k \neq 1)$ and not just $C \rightarrow 0$. Differentiating top and bottom separately is M0.	
	Note this is a hence and so attempts at e.g. use of the quotient rule scores M0. A1: cao and cso (May be un-simplified)	
	Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give the scores M1A0 if otherwise correct.	e same derivative so
(c)	B1: 25 only M1: Substitute $x = 4$ into their derived function M1: Uses their "25" and their "gradient" which has come from calculus (not the norma x = 4 to give correct ft equation of line. If using $y = mx + c$ must at least obtain a value f A1: any correct form e.g.	
	$y = -\frac{5}{2}x + 35, \qquad 5x + 2y - 70 = 0$	
	BUT NOT JUST $\frac{y-25}{x-4} = -\frac{5}{2}$, this scores M1A0	
	Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give the (c) and will lose the final A mark if otherwise correct.	correct answer in

Question Number	Scheme	Marks
13 (a)	$3kx^{2} + (8k+6)x + 9k - 2 = 0 \text{ or } 3kx^{2} + 8kx + 6x + 9k - 2 = 0$	B1
	Uses $b^2 - 4ac$ with $a = 3k$, $b = 8k \pm 6$ and $c = 9k \pm 2$	M1
	$-44k^2 + 120k + 36 < 0$ or $36 < 44k^2 - 120k$ o.e.	. 1
	Reached with no errors	A1
	$11k^2 - 30k - 9 > 0*$	A1*
		[4]
(b)	Attempts to solve $11k^2 - 30k - 9 = 0$ to give $k =$	M1
	\Rightarrow Critical values, $k = 3, -\frac{3}{11}$	A1
	$k > 3$ (or) $k < -\frac{3}{11}$	M1 A1cao
		[4]
	N 4	8 marks
	Notes	
(a)	B1: Multiplies by k and collects terms to one side in any order. Allow the x terms not to be contract the '= 0' may be implied by use of a correct discriminant. M1: Attempts $b^2 - 4ac$ with $a = 3k$, $b = 8k \pm 6$ and $c = 9k \pm 2$ or uses quadratic formula with	
	seen to solve their equation or uses $b^2 = 4ac$ or e.g. $b^2 < 4ac$. There must be no x's.	
	A1: Obtains a correct three term quadratic inequality that is not the printed answer with r A1: Correct answer with no errors	io errors seen.
(b)	M1: Uses factorisation, formula, or completion of square method to find two values for k or the correct answers with no obvious method for <u>the given</u> three term quadratic	finds two
	A1: Obtains $k = 3, -\frac{3}{11}$ accept awrt - 0.272	
	M1: Chooses outside region ($k <$ Their Lower Limit $k >$ Their Upper Limit) for a 3 term inequality. Do not award simply for diagram or table.	quadratic
	A1: $k > 3$ (or) $k < -\frac{3}{11}$ must be exact here but allow -0.27 for $-\frac{3}{11}$.	
	Allow other notation such as $\left(-\infty, -\frac{3}{11}\right) \cup (3, \infty)$	
	$k > 3$ and $k < -\frac{3}{11}$ and $-\frac{3}{11} > k > 3$ score M1A0	
	ISW if possible e.g. $k > 3$, $k < -\frac{3}{11}$ followed by $-\frac{3}{11} > k > 3$ can score M1A1	
	$k > 3$, $k > -\frac{3}{11}$ followed by $k > 3$ (or) $k < -\frac{3}{11}$ can score M1A1	
	Allow (b) to be solved in terms of x for the first 3 marks but the final A mark needs the region Fully correct answer with no working scores full marks. Answers that are otherwise correct but use $\leq \geq 1$ lose final mark.	ns in terms of <i>k</i> .

Question Number	Scheme	Marks
14(i)	$\log_{a} x + \log_{a} 3 = \log_{a} 27 - 1 \text{so} \log_{a} \frac{3x}{27} = -1$ Or $\log_{a} x + \log_{a} 3 = \log_{a} 27 - \log_{a} a \text{so} \log_{a} 3x = \log_{a} \frac{27}{a}$	M1 A1
	Or $\log_a x + 1 = \log_a 27 - \log_a 3 = \log_a 9$ so $\log_a ax = \log_a 9$	
	$\frac{3x}{27} = a^{-1}$	M1
	$x = 9a^{-1} \text{ or } \frac{9}{a}$	A1
		[4]
(ii)	$x^2 - 7x + 12 = 0$ and attempt to solve to give $x =$ or $\log_5 y =$ (implied by correct answers)	M1
	x (or $\log_5 y$) = 3 and 4	A1
	$y = 5^3$ or 5^4	dM1
	y = 125 and 625	A1
	y = 125 und 625	[4]
		8 marks
	Notes	
(i)	M1: Uses sum or difference of logs correctly e.g.	
	$\log x + \log 3 = \log 3x$ or $\log 27 - \log 3 = \log 9$ or $\log 27 - \log x = \log \frac{27}{x}$ etc.	
	or writes 1 as $\log_a a$	
	A1: Uses two rules correctly to obtain correct log equationM1: Removes logs correctly to obtain an equation connecting <i>x</i> and <i>a</i>A1: Correct simplified answer	
	M1: Removes logs correctly to obtain an equation connecting <i>x</i> and <i>a</i> A1: Correct simplified answer	l out of 4 if
	M1: Removes logs correctly to obtain an equation connecting x and a A1: Correct simplified answer Note that some candidates interpret $\log_a 27 - 1$ as $\log_a (27 - 1)$. This can score a maximum of 1 they have $\log x + \log 3 = \log 3x$	l out of 4 if
	M1: Removes logs correctly to obtain an equation connecting x and a A1: Correct simplified answer Note that some candidates interpret $\log_a 27 - 1$ as $\log_a (27 - 1)$. This can score a maximum of 1 they have $\log x + \log 3 = \log 3x$ Note that $\log_a x + \log_a 3 = \log_a 27 - 1$ so $\frac{\log_a 3x}{\log_a 27} = -1 \Rightarrow \frac{3x}{27} = a^{-1}$ etc. scores M1A0M0A0	
	M1: Removes logs correctly to obtain an equation connecting x and a A1: Correct simplified answer Note that some candidates interpret $\log_a 27 - 1$ as $\log_a (27 - 1)$. This can score a maximum of 1 they have $\log x + \log 3 = \log 3x$	
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Question Number	Scheme	Marks
15 (a)	Mid-point of $AB = (2, -3)$	M1 A1
	$(r^{2}) = (12 - "2")^{2} + (2 - "-3")^{2}$ or $(r^{2}) = (-8 - "2")^{2} + (-8 - "-3")^{2}$ or $(d^{2}) = (-8 - 12)^{2} + (-8 - 2)^{2}$	M1
	$r^2 = 125$	A1
	$"125" = (x \pm "2")^{2} + (y \pm "-3")^{2}$	M1
	$125 = (x-2)^2 + (y+3)^2$	A1
		[6]
(b)	gradient from "(2, -3)" to (4, 8) = $\frac{8 - "-3"}{4 - "2"}$, $\left(=\frac{11}{2}\right)$	M1
	ZM has gradient $-\frac{1}{m}$ $\left(=-\frac{2}{11}\right)$	M1
	Either: $y - 8 = "-\frac{2}{11}"(x-4)$ or: $y = "-\frac{2}{11}"x + c$ and $8 = "-\frac{2}{11}"(4) + c \implies c = "8\frac{8}{11}"$	ddM1
	2x + 11y - 96 = 0	A1
		[4]
		(10marks)
	Notes	(Iomarks)
	M1: Finds radius or radius ² , diameter or diameter ² using any valid method – probably distance from centre to one of the points. Need not state $r = \dots$ so ignore lhs – you are just looking for correct use of Pythagoras with or without the square root so ignore how they reference it for this mark. A1: for any equivalent $r^2 = 125$ or $r = \sqrt{125}$ (11.18) etc. Their numeric answer must be identified here as either r or r^2 (may be implied by their equation). If they halve it or double it, this is M1 A0. M1: Attempt to use a true equation for circle with their centre and radius or the letter r , allow sign slips in brackets but do not allow use or r instead of r^2 in the equation. So must be using $r^2 = (\mathbf{x} \pm)^2 + (\mathbf{y} \pm)^2$ A1: correct answer only (Allow $(5\sqrt{5})^2$ instead of 125 but not $5\sqrt{5}^2$)	
(b)	 M1: States or uses gradient equation correctly with their centre and (4, 8). Must be using the (4, 8). If no method is shown and gradient incorrect for their values score M0. M1: Finds negative reciprocal. Follow through their gradient ddM1: Correct straight line method with (4, 8) and perpendicular gradient. Dependent on bot method marks having been scored. A1: cao – accept multiples of this equation (Note integer coefficients not required) 	
	A common error here is to use the diameter to find the gradient. This usually scores M0M1dd just one mark for the perpendicular gradient rule. (b) Alternative uses implicit differentiation: e.g.	
	A common error here is to use the diameter to find the gradient. This usually scores M0M1dd just one mark for the perpendicular gradient rule.	

Question Number	Scheme	Marks
16(a)	$\frac{1}{2}x + 1 = x^2 - 4x + 3$	M1
-	$2x^2 - 9x + 4 = 0 \implies x = \frac{1}{2} \text{ or } x = 4$	dM1 A1
	y = 5/4 or $y = 3$	dM1 A1
		[5]
(b)	Curve meets x-axis at $x = 3$ and at $x = 1$ (No need to see $y = 0$)	M1 A1 [2]
	NOTE that the subscripted A's refer to areas on the diagram given at the end of the scheme. All the method marks are for <u>their</u> $x = 1/2$, 4, 1 and 3	
(c) Way 1	$\int x^2 - 4x + 3 \mathrm{d}x = \frac{1}{3}x^3 - 2x^2 + 3x$	M1 A1
•	Use limits 1 and $\frac{1}{2} \left[\left(\frac{1}{3}(1)^3 - 2(1)^2 + 3 \times 1 \right) - \left(\frac{1}{3} \left(\frac{1}{2} \right)^3 - 2 \cdot \left(\frac{1}{2} \right)^2 + 3 \times \left(\frac{1}{2} \right) \right) \right] A_1$	M1
-	Use limits 4 and 3 $[(\frac{1}{3}(4)^3 - 2(4)^2 + 3 \times (4)) - (\frac{1}{3}(3)^3 - 2(3)^2 + 3 \times (3))] A_2$	M1
-	Area of trapezium =	
	$\frac{1}{2}(a+b) \times h = \frac{1}{2}(\frac{5}{4}+3) \times (4-\frac{1}{2}) = \dots \text{ or } \int_{\frac{1}{2}}^{4} (\frac{1}{2}x+1) dx = \left[\frac{1}{4}x^2 + x\right]_{\frac{1}{2}}^{4} = (4+4) - (\frac{1}{16}+\frac{1}{2}) = \dots$	M1
	7.4375 $(7\frac{7}{16})$ $(\frac{119}{16})$ (may be implied by correct final answer)	A1
-	Uses correct combination of correct areas. Area of region = Area of trapezium $-A_1 - A_2$ Dependent on all previous method marks	ddddM1
	$= 7.4375 - \frac{7}{24} - \frac{4}{3} = \frac{93}{16} \text{ or } 5.8125$	A1
(a)	Alternative method using filing any we' and subtracting area below a suic	[8]
(c) Way 2	Alternative method using "line – curve" and subtracting area below x- axis $\int -x^2 + \frac{9}{2}x - 2dx = -\frac{x^3}{3} + \frac{9}{4}x^2 - 2x \text{ or } \int x^2 - \frac{9}{2}x + 2dx = \frac{x^3}{3} - \frac{9}{4}x^2 + 2x$	M1A1
-	Use limits $\frac{1}{2}$ and 4 on this <i>subtracted</i> integration $(A_3 + A_4 + A_5 + A_6) = 6\frac{2}{3} + \frac{23}{48} = \dots$	M1
-	$\pm \int x^2 - 4x + 3dx = \pm \left(\frac{1}{3}x^3 - 2x^2 + 3x\right)$	M1
	Use limits 1 and 3 on their integrated curve to obtain $A_6 = \pm \frac{4}{3}$	M1A1
-	Uses correct combination of correct areas. Area of region = $(A_3 + A_4 + A_5 + A_6) - A_6$ Dependent on all previous method marks	ddddM1
	$6\frac{2}{3} + \frac{23}{48} - \frac{4}{3} = \frac{93}{16}$	A1
		[8]
(c) Way 3	Alternative method using "line – curve" for areas A_3 and A_4 and adding smaller trapezium $\int -x^2 + \frac{9}{2}x - 2dx = -\frac{x^3}{3} + \frac{9}{4}x^2 - 2x \text{ or } \int x^2 - \frac{9}{2}x + 2dx = \frac{x^3}{3} - \frac{9}{4}x^2 + 2x$	M1A1
-	Use limits 1 and $\frac{1}{2} \left[\left(-\frac{1}{3}(1)^3 + \frac{9}{4}(1)^2 - 2 \times 1 \right) - \left(-\frac{1}{3}(\frac{1}{2})^3 + \frac{9}{4}(\frac{1}{2})^2 - 2 \times \frac{1}{2} \right] A_3$	M1
	Use limits 4 and 3 $\left[\left(-\frac{1}{3}(4)^3 + \frac{9}{4}(4)^2 - 2 \times 4\right) - \left(-\frac{1}{3}(3)^3 + \frac{9}{4}(3)^2 - 2 \times 3\right] A_4$	M1
	Area of trapezium =	
	$\frac{1}{2}(a+b) \times h = \frac{1}{2}(\frac{3}{2} + \frac{5}{2}) \times (3-1) = \dots \text{ or } \int_{1}^{3} (\frac{1}{2}x+1)dx = \left[\frac{1}{4}x^{2} + x\right]_{1}^{3} = (\frac{9}{4}+3) - (\frac{1}{4}+1) = \dots$	M1
	= 4	A1
	Uses correct combination of correct areas. Area of region $= A_3 + A_4 + A_5$ Dependent on all previous method marks	ddddM1
L		

(c) Way 4	Alternative method: Finds area of larger trapezium and subtracts A ₁ + A ₂ which is found by integrating quadratic between ¹ / ₂ and 4 and adding area below <i>x</i> -axis	
·	$\int x^2 - 4x + 3 \mathrm{d}x = \frac{1}{3}x^3 - 2x^2 + 3x$	M1 A1
	Use limits 4 and $\frac{1}{2} \left[\left(\frac{1}{3} (4)^3 - 2(4)^2 + 3 \times 4 \right) - \left(\frac{1}{3} \left(\frac{1}{2} \right)^3 - 2 \cdot \left(\frac{1}{2} \right)^2 + 3 \times \left(\frac{1}{2} \right) \right) \right] A_1 + A_2 - A_6$	M2
	AND Use limits 3 and 1 $\pm [(\frac{1}{3}(3)^3 - 2(3)^2 + 3 \times 3) - (\frac{1}{3}(1)^3 - 2.(1)^2 + 3 \times (1))] \pm A_6$	1112
	Area of trapezium = $\frac{4}{4}$	
	$\frac{1}{2}(a+b) \times h = \frac{1}{2}(\frac{5}{4}+3) \times (4-\frac{1}{2}) = \dots \text{ or } \int_{\frac{1}{2}}^{\pi} (\frac{1}{2}x+1) dx = \left[\frac{1}{4}x^2 + x\right]_{\frac{1}{2}}^{4} = (4+4) - (\frac{1}{16}+\frac{1}{2}) = \dots$	M1
	7.4375 $(7\frac{7}{16})$ (may be implied by correct final answer)	A1
	Uses correct combination of correct areas. Area of region = $7.4375 - (A_1 + A_2 - A_6 + A_6)$	ddddM1
	Dependent on all previous method marks	ddddivi i
	$= 7.4375 - \left(\frac{7}{24} + \frac{4}{3}\right) = \frac{93}{16}$	A1
		[8]
	N. 4	15 marks
	Notes M1: Puts equations equal or finds x in terms of y and substitutes or substitutes for x	
(a)	dM1: Solves three term quadratic in x to obtain $x =$ or in y to obtain $y =$ (Dependent on fi A1: Both answers correct dM1: Obtains at least one value for y or x (Dependent on first M)	rst M)
	A1: Both correct Note: Allow candidates to obtain $x^2 - \frac{9}{2}x + 2 = 0$ and solve as $(2x-1)(x-4) = 0 \Rightarrow x = \frac{1}{2}, 4$	
	The coordinates do not need to be 'paired'	
(b)	M1: Attempts to solve $0 = x^2 - 4x + 3$ according to the usual rules A1: cao Attempts by T&I can score both marks for $x = 1$ and $x = 3$. If one solution is obtained by this, so	oro M140
	For (c) do not allow 'mixed' methods. For their strategy, they must be finding the appropute of the scheme that gives the most credit for the candidate of the scheme that gives the most credit for the scheme that gives the most credit for the scheme the scheme the scheme that gives the most credit for the scheme the scheme the scheme that gives the most credit for the scheme the sche	priate areas
(c) Way 1	M1: Attempt at integration of the given quadratic expression $(x^n \rightarrow x^{n+1} \text{ at least once})$	
	A1: Correct integration of the given quadratic expressionM1: Finds area of A1	
	M1: Finds area of A_2	
	M1: Finds area of appropriate trapezium $7.4275(7.7)$	
	A1: Correct area of trapezium 7.4375 $(7\frac{7}{16})$ ddddM1: correct final combination	
	A1: any correct form of this exact answer	
(c) Way 2	M1: Attempt at integration of \pm (the given quadratic expression – the given line) $(x^n \rightarrow x^{n+1})$ at	t least once)
Way 2	 A1: Correct integration as shown in the mark scheme. Allow correct answer even if terms not collected nor simplified. If there are sign errors when subtracting before valid attempt at integration, score M1A0 M1: Uses the limits ¹/₂ and 4 on their <i>subtracted</i> integration M1: Attempts to integrate curve 	
	M1: Uses the limits 1 and 3 on the integrated curve C	
	A1: Obtains $A_6 = \pm \frac{4}{3}$	
	ddddM1: correct final combination A1: any correct form of this exact answer	
	Note: A common error with this method is to use the limits ¹ / ₂ and 4 on their <i>subtracted</i> integrat	ion and then
	stop (this should give an area of $\frac{343}{48}$). This will usually score 3/8 in (c)	

(c) Way 3	M1: Attempt at integration of ±(the given quadratic expression – the given line) $(x^n \rightarrow x^{n+1} \text{ at least once})$
Way 3	A1: Correct integration as shown in the mark scheme. Allow correct answer even if terms not collected nor
	simplified. If there are sign errors when subtracting before valid attempt at integration, score M1A0
	M1: Uses the limits $\frac{1}{2}$ and 1 on their <i>subtracted</i> integration
	M1: Uses the limits 4 and 3 on their <i>subtracted</i> integration
	M1: Finds area of appropriate trapezium
	A1: Correct area of trapezium 4
	ddddM1: correct final combination
	A1: any correct form of this exact answer
(c) Way 4	M1: Attempt at integration of the given quadratic expression $(x^n \rightarrow x^{n+1} \text{ at least once})$
Way I	A1: Correct integration of the given quadratic expression
	M2: Finds area of $A_1 + A_2 - A_6$ by using the limits $\frac{1}{2}$ and 4 and finds area of A_6 by using the limits 1 and 3
	M1: Finds area of appropriate trapezium
	A1: Correct area of trapezium 7.4375 $(7\frac{7}{16})$
	ddddM1: correct final combination
	A1: any correct form of this exact answer

Diagram for Question 16

